A dispersively accurate compact finite difference method for the Degasperis-Procesi equation

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Abstract:

In this paper we are aimed to solve the non-dissipative Degasperis-Procesi equation based on the $u - P$ formulation. To resolve the computational difficulty at the wave crest where the first-order derivative may diverge and the shockpeakon solution may form, the first-order spatial derivative term in the two-step equations will be approximated in a conservative form. The resulting equations will be approximated by the symplecticity-preserving time-stepping scheme and the spatial discretization scheme that can optimize the numerical wavenumber for the first-order spatial derivative term. This scheme will be developed in a three-point grid stencil with the accuracy order of seventh within the combined compact finite difference framework. Besides the validation of numerical accuracy, we will in particular address the discrete conservation of Hamiltonians even when peakon collides with antipeakon and generates, as a result, a shockpeakon. We will also demonstrate the capability of applying the proposed numerical method to sharply resolve some important features of the third-order dispersive DP equation.

keywords: non-dissipative; Degasperis-Procesi equation; shockpeakon; symplecticity-preserving; conservation of Hamiltonians.